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Numerical analysis of combined radiation and unsteady natural convection within a horizontal annular space

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Abstract *The effect of radiation on unsteady natural convection in a two-dimensional participating medium between two horizontal concentric and vertically eccentric cylinders is investigated numerically. The equations of transfer are written by using a bicylindrical coordinates system, the stream function, and the vorticity. The finite volume radiation solution method and the control volume approach are used to discretize the coupled equations of radiative transfer, momentum, and energy. Original results are obtained for three eccentricities, Rayleigh number equal to* 10^4, 10^5*, and a wide range of radiation-conduction parameter. The effects of optical thickness, wall emissivity, and scattering on flow intensity and heat transfer are discussed.*

Nomenclature

e	= dimensionless eccentricity, $e = \frac{\overline{o_i o_o}}{r_o - r_i}$
g	= gravitational acceleration: $m.s^{-2}$
I	= radiant intensity: $W.m^{-2}.sr^{-1}$
I^0	= blackbody intensity: $W.m^{-2}.sr^{-1}$
L	= total number of discrete solid angles
L_+	= total number of discrete solid angles oriented to a given boundary
N	= dimensionless quantity, $N_i^1 = \frac{1}{\Delta\Omega^1} \int_{\Delta\Omega^1} \Omega.\mathbf{n}_i d\Omega$
N_θ, N_φ	= number of discrete angles in polar θ and azimuthal φ directions
N_η, N_ξ	= number of nodes in η and ξ directions
n	= unit vector normal to the control volume face
o	= centre of a cross section of the cylinder (Figure 1)
Pr	= Prandtl number, $Pr = \nu/\alpha$
Q	= dimensionless total heat flux defined in equation (25)
Q_c	= dimensionless conductive heat flux defined in equation (23)
Q_r	= dimensionless radiative heat flux defined in equation (24)
r	= radius of a cross section of the cylinder (Figure 1): m
R	= source term defined in equation (12): $W.m^{-2}.sr^{-1}$
Ra	= Rayleigh number, $R_a = g\beta_c (r_o - r_i)^3 (T_i - T_o)/\alpha\nu$
Rc	= radiation-conduction parameter, $R_c = r_i \sigma T_o^3/\lambda$
s	= distance in the direction Ω of the intensity – m
t	= dimensionless time, $t = t'\alpha/(r_o - r_i)^2$
T	= dimensionless temperature, $T = (T' - T_o)/(T_i - T_o)$
u_η	= dimensionless η-component of velocity, $u_\eta = u_\eta'(r_o - r_i)/\alpha = \frac{1}{C}\frac{\partial\psi}{\partial\xi}$
u_ξ	= dimensionless ξ-component of velocity, $u_\xi = u_\xi'(r_o - r_i)/\alpha = -\frac{1}{C}\frac{\partial\psi}{\partial\eta}$

Greek symbols

α = thermal diffusivity: m2.s^{-1}
β = extinction coefficient: m^{-1}
β_c = coefficient of thermal expansion: m^{-1}
ΔA = area of a control volume face: m^2
Δv = control volume: Ωm^3
$\Delta\Omega$ = control solid angle, $\Delta\Omega^1 = \int_{\theta 1-}^{\theta 1+}\int_{\varphi 1-}^{\varphi 1+} \sin\theta d\theta d\varphi$: sr
ε = emissivity
Φ = temperature ratio, $\Phi = (T_i - T_o)/T_o$
Γ = radius ratio, $\Gamma = r_o/r_i$
η, ξ, z = bicylindrical coordinates
η_i = boundary of the inner cylinder in bicylindrical coordinates system, $\eta_i = \cosh^{-1}\left[\frac{1-e^2+\Gamma(1+e^2)}{2\Gamma e}\right]$
η_o = boundary of the outer cylinder in bicylindrical coordinates system, $\eta_o = \cosh^{-1}\left[\frac{1+e^2+\Gamma(1+e^2)}{2e}\right]$
λ = thermal conductivity: W.m^{-1}.K^{-1}
ν = kinematic viscosity: m^2.s^{-1}
θ, φ = polar and azimuthal angles respectively: rd
ρ = density: kg.m^{-3}
σ = Stefan-Boltzmann constant: W.m^{-2}.K^{-4}
τ = optical thickness, $\tau = \beta r_i$
ω = dimensionless vorticity, $\omega = \omega'(r_o - r_i)^2/\alpha$
ω_0 = scattering albedo
Ω = unit vector in the direction of the intensity
ψ = dimensionless stream function, $\psi = \psi'/\alpha$

Subscripts

e, w, n, s = faces of control volume centred in P
E,W, N, S = nodes around the nodal point P
i = inner cylinder or control volume face
o = outer cylinder
P = nodal point
W = boundary of the computational domain

Superscripts

l, l' = discrete angular directions

Introduction

The problem of natural convection heat transfer in the annulus between two concentric and eccentric horizontal cylinders has been a subject of intensive research in the recent years. Comprehensive reviews (Powe *et al.*, 1971; Kuehn and Goldstein, 1976; 1978; Charrier-Mojtabi *et al.*, 1979; Projahn *et al.*, 1981; Tsui and Tremblay, 1984; Karki and Patankar, 1988; Kumar, 1988) on natural convection in such configuration are available and there is no need to repeat them.

Over the last three decades, combined radiation and natural convection in enclosure has received considerable attention. Larson and Viskanta (1976) treated transient combined laminar free convection and radiation in a rectangular enclosure. To study the problem of radiation-natural convection in square enclosures with vertical partitions, Chang *et al.* (1983) developed a modified radial flux method. Hassab and Ozizik (1979), Lauriat and Desreyaud (1985) then Fusegi and Farouk (1989) utilised the P1 approximation to model the radiation-natural convection interactions within a rectangular enclosure. The first two references analysed the effect of radiation and boundary conditions on the stability of the fluid, and the last examined both the laminar and turbulent regimes. Webb and Viskanta (1987) analysed natural convection induced by irradiation in a rectangular participating medium using one dimensional radiation model. Yucel *et al.* (1989) used the discrete ordinate method to study radiation-natural convection in inclined enclosure filled with a heat generating fluid. More recently, Tan and Howell (1991) adapted the product-integral method to solve the radiative transfer equation in radiation-natural convection problem within a two-dimensional square enclosure.

To our knowledge, the only publications treating the radiation-natural convection problem within a horizontal annular space are those of Onyegegbu (1986), Tan and Howell (1989), and Borjini *et al.* (1998). All of them assumed a steady flow. The first two references considered the case of concentric cylinders and utilised, to solve the radiative transfer equation, the Milne-Eddington approximation and the YIX technique respectively. The last reference examined the case of eccentric cylinders using the finite volume method. This method was initially developed by Raithby and Chui (1990) and Chai *et al.* (1994). Chui *et al.* (1992) implemented it to study the radiative transfer within cylindrical enclosures. In order to study irregular geometries, Chai *et al.* (1995) developed it in curvilinear coordinates system. Moder *et al.* (1996) used it to study axisymmetric radiative transfer through a cylinder and nonaxisymmetric radiative transfer through two-and three-dimensional annular sectors. Lee *et al.* (1996) utilised it to analyse radiative-convective heat transfer around a circular cylinder in a cross flow. Kim and Baek (1997) applied it to study radiative transfer in axisymmetric as well as three-dimensional cylindrical geometries.

This work presents a study of radiation-unsteady natural convection interactions within horizontal annular space between concentric and vertically eccentric cylinders.

Formulation and method of resolution

As shown in Figure 1, the physical system consists of an annulus bounded by two horizontal isothermal cylinders. The cylinder walls are grey, diffuse, and have the same emissivity. The annular medium is considered to be grey, emitting, absorbing, and isotropically scattering gas. Initially, the annulus is at a uniform temperature T_o, while the temperature of the inner cylinder is suddenly changed to a higher temperature T_i and the outer cylinder is maintained at T_o. It is assumed that the flow in the system is laminar with no-slip conditions

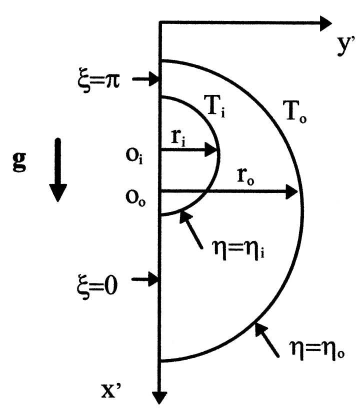


Figure 1.
Schematic representation of the system section

applicable at the walls. The viscous dissipation is negligible, the physical properties are constant, and the Boussinesq approximation is valid.

Using bicylindrical coordinates system (η, ξ, z) (Moon and Spencer, 1971) and a scaling length, velocity, and time by $r_o - r_i$, $\alpha/(r_o - r_i)$ and $(r_o - r_i)^2/\alpha$, the governing equations in dimensionless stream function-vorticity form are:

$$\frac{1}{C^2}\left(\frac{\partial^2\psi}{\partial\eta^2} + \frac{\partial^2\psi}{\partial\xi^2}\right) = -\omega \tag{1}$$

$$\frac{\partial\omega}{\partial t} + \frac{\partial}{\partial\eta}\left(\mathrm{C}u_\eta\omega - \mathrm{Pr}\frac{\partial\omega}{\partial\eta}\right) + \frac{\partial}{\partial\xi}\left(\mathrm{C}u_\xi\omega - \mathrm{Pr}\frac{\partial\omega}{\partial\xi}\right) = \mathrm{C\,Pr\,Ra}\left[\mathrm{B}\frac{\partial \mathrm{T}}{\partial\eta} - \mathrm{A}\frac{\partial \mathrm{T}}{\partial\xi}\right] \tag{2}$$

$$\frac{\partial \mathrm{T}}{\partial t} + \frac{\partial}{\partial\eta}\left(\mathrm{C}u_\eta \mathrm{T} - \frac{\partial \mathrm{T}}{\partial\eta}\right) + \frac{\partial}{\partial\xi}\left(\mathrm{C}u_\xi \mathrm{T} - \frac{\partial \mathrm{T}}{\partial\xi}\right) = \frac{\mathrm{C}^2(\Gamma - 1)Rc(1-\omega_0)\tau}{\Phi}\left[\frac{1}{\sigma \mathrm{T}_o^4}\int \mathrm{I}d\Omega - 4(1+\Phi \mathrm{T})^4\right] \tag{3}$$

where:

$$\mathrm{A}(\eta,\xi) = \frac{1 - \cosh\eta\cos\xi}{\cosh\eta - \cos\xi} \tag{4}$$

$$\mathrm{B}(\eta,\xi) = \frac{\sinh\eta\sin\xi}{\cosh\eta - \cos\xi} \tag{5}$$

$$\mathrm{C}(\eta,\xi) = \frac{1}{r_o - r_i}\frac{r_i\sinh\eta_i}{\cosh\eta - \cos\xi} \tag{6}$$

The associated initial and boundary conditions for the problem considered are:

(1) for $t \leq 0$

$$\omega = \psi = \frac{\partial\psi}{\partial\eta} = \frac{\partial\psi}{\partial\theta} = \mathrm{T} = 0 \qquad \text{everywhere} \tag{7}$$

(2) for $t > 0$

- on the inner cylinder $(\eta = \eta_i; 0 \leq \xi \leq \pi)$:

$$\psi = \frac{\partial\psi}{\partial\eta} = 0,\ \ \mathrm{T} = 1 \tag{8}$$

- on the outer cylinder $(\eta = \eta_o; 0 \leq \xi \leq \pi)$:

$$\psi = \frac{\partial\psi}{\partial\eta} = \ \mathrm{T} = 0 \tag{9}$$

- on the symmetry axis ($\xi = 0, \pi; \eta_o < \eta < \eta_i$):

$$\psi = \omega = \frac{\partial T}{\partial \xi} = 0 \tag{10}$$

The radiative transfer equation, in absorbing, emitting, and isotropically scattering medium, can be written as (Siegel and Howell, 1992):

$$\frac{\partial \mathrm{I}(\mathrm{s}, \Omega)}{\partial \mathrm{s}} + \beta \mathrm{I}(\mathrm{s}, \Omega) = \beta \mathrm{R} \tag{11}$$

where:

$$\mathrm{R} = (1 - \omega_0)\mathrm{I}^0(\mathrm{s}) + \frac{\omega_0}{4\pi} \int_{4\pi} \mathrm{I}(\mathrm{s}, \Omega')\mathrm{d}\Omega' \tag{12}$$

Where I^0 is the blackbody intensity, s is the distance in the direction of the intensity, Ω and Ω' are the unit vectors in the outward and the inward directions of radiation, $d\Omega'$ is the elementary control solid angle centred on Ω', and ω_0 is the scattering albedo.

The finite volume method is utilised to discretize equation (11). The computational domain is divided into finite volumes and the intensity direction into finite number of solid angles. This equation is integrated over each control volume and control angle. Using the step scheme, one gets the following system of algebraic coupled equations:

$$\mathrm{a}_\mathrm{p}^\mathrm{l}\mathrm{I}_\mathrm{p}^\mathrm{l} = \mathrm{a}_\mathrm{W}^\mathrm{l}\mathrm{I}_\mathrm{W}^\mathrm{l} + \mathrm{a}_\mathrm{E}^\mathrm{l}\mathrm{I}_\mathrm{E}^\mathrm{l} + \mathrm{a}_\mathrm{S}^\mathrm{l}\mathrm{I}_\mathrm{S}^\mathrm{l} + \mathrm{a}_\mathrm{N}^\mathrm{l}\mathrm{I}_\mathrm{N}^\mathrm{l} + \mathrm{b}_\mathrm{P} \tag{13}$$

where:

$$\mathrm{a}_\mathrm{W}^\mathrm{l} = \Delta \mathrm{A}_\mathrm{w} \max\left[-\mathrm{N}_\mathrm{w}^\mathrm{l}, 0\right] \tag{14}$$

$$\mathrm{a}_\mathrm{E}^\mathrm{l} = \Delta \mathrm{A}_\mathrm{e} \max\left[-\mathrm{N}_\mathrm{e}^\mathrm{l}, 0\right] \tag{15}$$

$$\mathrm{a}_\mathrm{N}^\mathrm{l} = \Delta \mathrm{A}_\mathrm{n} \max\left[-\mathrm{N}_\mathrm{n}^\mathrm{l}, 0\right] \tag{16}$$

$$\mathrm{a}_\mathrm{S}^\mathrm{l} = \Delta \mathrm{A}_\mathrm{s} \max\left[-\mathrm{N}_\mathrm{s}^\mathrm{l}, 0\right] \tag{17}$$

$$\begin{aligned} \mathrm{a}_\mathrm{p}^\mathrm{l} =& \Delta \mathrm{A}_\mathrm{w} \max\left[\mathrm{N}_\mathrm{w}^\mathrm{l}, 0\right] + \Delta \mathrm{A}_\mathrm{e} \max\left[\mathrm{N}_\mathrm{e}^\mathrm{l}, 0\right] + \Delta \mathrm{A}_\mathrm{s} \max\left[\mathrm{N}_\mathrm{s}^\mathrm{l}, 0\right] \\ &+ \Delta \mathrm{A}_\mathrm{n} \max\left[\mathrm{N}_\mathrm{n}^\mathrm{l}, 0\right] + \beta \Delta \mathrm{v}_\mathrm{p} \end{aligned} \tag{18}$$

$$\mathrm{b}_\mathrm{P} = \beta \mathrm{R}_\mathrm{p} \Delta \mathrm{v}_\mathrm{p} \tag{19}$$

More details are in Chai *et al.* (1994; 1995) and Borjini *et al.* (1998).

For the grey and diffuse surface, the radiative boundary conditions are:

$$I_W^l = \frac{\varepsilon\sigma T_W^4}{\pi} + \frac{(1-\varepsilon)}{\pi}\sum_{L_+} |N_W^{l'}| I_W^{l'} \Delta\Omega^{l'} \tag{20}$$

The symmetry axis is assumed to behave like a grey and specular surface (Kim and Baek, 1991). Thus the correspondent boundary conditions can be written as:

$$I_W^l = I_W^{l'} \quad \text{if} \quad \Omega^l.\mathbf{e}_\eta = \Omega^{l'}.\mathbf{e}_\eta \quad \text{and} \quad \Omega^l.\mathbf{e}_\xi > 0, \xi = 0 \tag{21}$$

$$I_W^l = I_W^{l'} \quad \text{if} \quad \Omega^l.\mathbf{e}_\eta = \Omega^{l'}.\mathbf{e}_\eta \quad \text{and} \quad \Omega^l.\mathbf{e}_\xi < 0, \xi = \pi \tag{22}$$

We define the following dimensionless fluxes:

- local conductive flux:

$$Q_c = \frac{1}{C}\frac{\partial T}{\partial \eta} \tag{23}$$

- local radiative flux:

$$Q_r = \frac{Rc}{\Phi\sigma T_o^4}\sum_{l=1}^{L} N^l I^l \Delta\Omega^l \tag{24}$$

- local total flux:

$$Q = Q_c + Q_r \tag{25}$$

- average conductive flux:

$$\bar{Q}_c = \int_0^\pi CQ_c d\xi \Big/ \int_0^\pi Cd\xi \tag{26}$$

- average radiative flux:

$$\bar{Q}_r = \int_0^\pi CQ_r d\xi \Big/ \int_0^\pi Cd\xi \tag{27}$$

- average total flux:

$$\bar{Q} = \bar{Q}_c + \bar{Q}_r \tag{28}$$

Equations (1-3) are discretized using the control volume method (Patankar, 1980). The power-low scheme (Patankar, 1980) for treating convective terms and the fully implicit procedure to discretize the temporary derivatives are retained. The grid is uniform in both directions with additional nodes on boundaries. The resulting nonlinear algebraic equations are solved using the successive relaxation iterating scheme (Bejan, 1984). The equation of radiative transfer is solved by repeatedly sweeping across the grid until convergence without taking into account the optimal order in which the nodes should be visited. The solution is considered acceptable when the following convergence criterion is satisfied for each step of time:

$$\frac{\max|\psi^{n} - \psi^{n-1}|}{\max|\psi^{n}|} + \max|T^{n} - T^{n-1}| \leq 10^{-5} \tag{29}$$

where the superscript n designates the nth iteration.

The solution is regarded as a steady state if the following criterion is satisfied:

$$\frac{\max|\psi^{m} - \psi^{m-1}|}{\max|\psi^{m}|} \leq 10^{-5}$$

where m denotes the number of the time step. We define the time ts which characterises the beginning of this state by the smallest instant verifying this condition.

We utilise the spatial and angular mesh chosen by Borjini *et al.* (1998): $N_\eta \times N\xi = 31 \times 61$ and $N_\theta \times N\varphi = 2 \times 12$. Figure 2 shows the influence of the refinement of time increment on inner and outer average fluxes when the inner cylinder is moved toward the bottom (e = –0.625) with Ra = 10^5 and Rc = 0. When the time step is weak, the time to reach the steady solution is long. Indeed, the practice of solving a steady-state problem via the unsteady formulation is simply a particular kind of under-relaxation procedure (Patankar, 1980). The smaller is the time step chosen, the stronger is the under-relaxation. The step $\Delta t = 10^{-4}$ is retained to carry out all numerical tests. Refining this time step results in minor changes of the transient patterns and has no detectable effect on the steady solution. To validate the present numerical code, steady natural convection between concentric and vertically eccentric cylinders was solved and compared with the corresponding benchmark solution (Kuehn and Goldstein, 1976; 1978 ; Projahn *et al.*, 1981). The comparison shows quite good agreement with both experimental and numerical results.

Results and discussion

In the absence of contrary indication, all the results are obtained with $\Phi = 1$, Pr = 0.7, $\Gamma = 2.6$, $\tau = 1$, $\omega_0 = 0$, and $\varepsilon = 1$.

Figure 3 shows the time variations of streamline and temperature contours for $Ra = 10^4$, Rc = 0, 1, and 10 when the inner cylinder is moved towards the bottom (e = –0.625). Because of the action at distance of radiation, the fluid far from the hot wall is more quickly heated when the radiation-conduction parameter increases and the ascendant movement, when t increases, of the

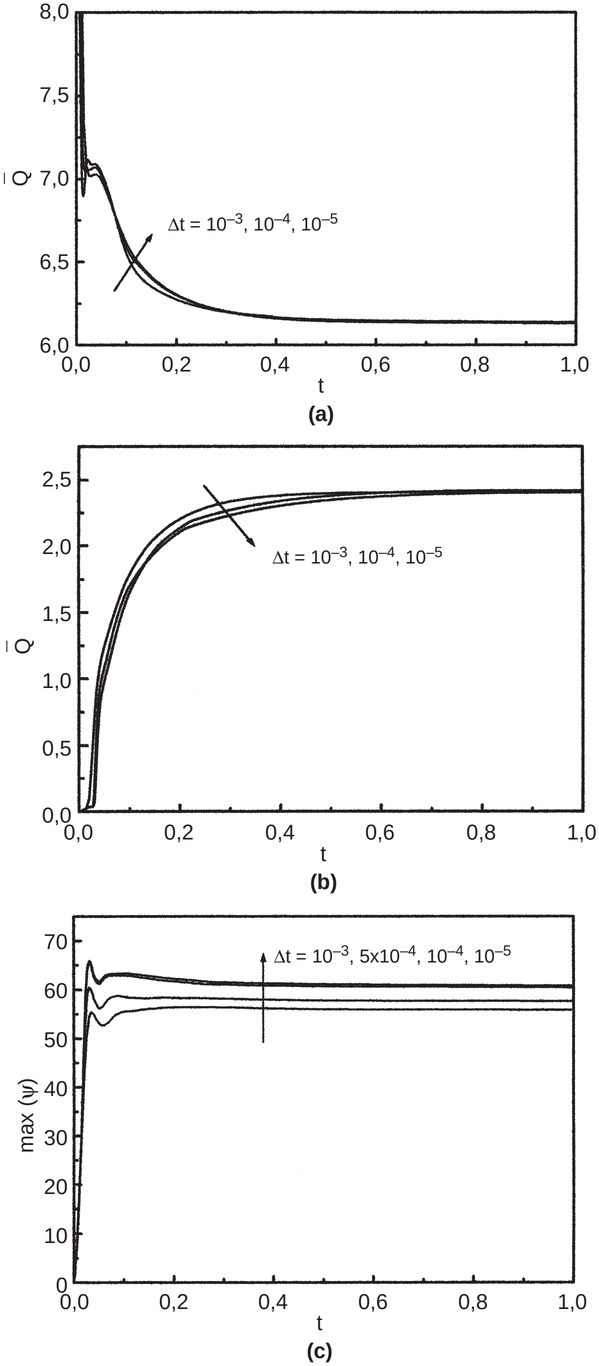


Figure 2.
Influence of step time refinement for $e = -0.625$, $Ra = 10^5$, and $Rc = 0$ on: (a) average total inner flux; (b) average total outer flux; (c) maximum of the stream function

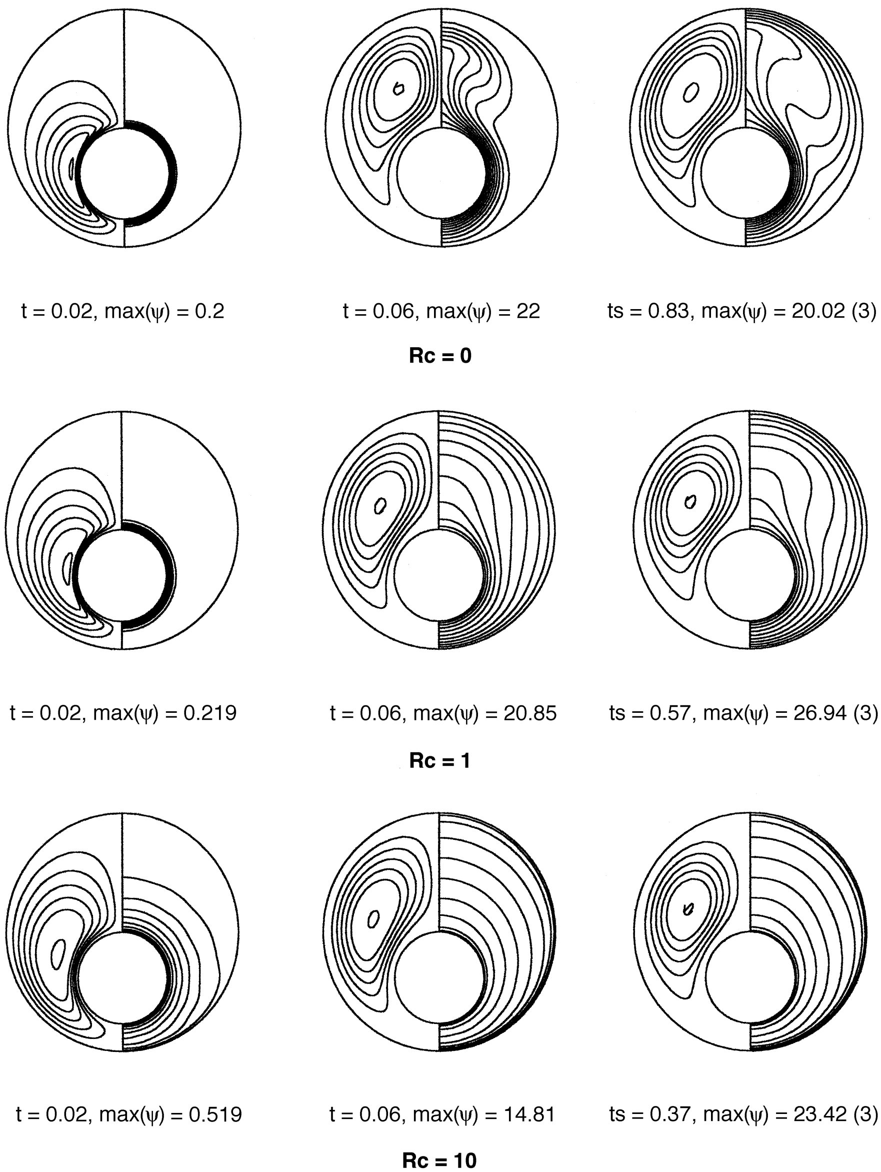


Figure 3.
Influence of the radiation-conduction parameter on streamline and temperature contours for e = –0.625 and Ra = 10^4

position of the core diminishes. For Rc = 10, the steady thermal field is fast reached, while the flow still manifests a transient zone. Similar results are observed for $\mathrm{Ra} = 10^5$ as well as for $\mathrm{Ra} = 10^4$ in the concentric case and in the one in which the inner cylinder is moved toward the top (e = 0.625).

Henceforth we only consider the values of Rc between 0 and 1. For higher values, heat is essentially transferred by radiation (greater than 80 per cent of the total heat transfer when Rc equals 10).

Figure 4 depicts the influence of radiation-conduction parameter on the time evolution of the maximum of stream function when $Ra = 10^4$ and e = –0.625. It shows that the stream function increases at first until reaching a maximum and decreases at larger times; i.e. the effect of the local Rayleigh number on the flow pattern decreases with time due to mixing and internal temperature balancing (Küblbeck *et al.*, 1980). The radiative transfer accelerates this balancing. When Rc increases, the maximum of $\max\psi$, situated at $t \approx 0.095$, becomes less pronounced, whereas the asymptotic limit amplifies: $\max\psi$ increases by 35 per cent when Rc varies from 0 to 1. This amplification also exists for steady heat transfer: the average total flux decreases by 82 per cent when Rc varies from 1 to 0. This is confirmed by Figure 5a which also shows that, when radiation is dominating, heat transfer, particularly on the outer cylinder, varies regularly.

According to Figure 5b, we deduce that the percentage of radiative flux decreases with time on outer cylinder and increases on inner cylinder. In the transient zone, radiative transfer is more important on outer cylinder than on inner cylinder and conversely in the steady zone. This is explained on the one hand by the choice of the boundary and the initial conditions, which impose, at t = 0, a nil conductive flux on outer cylinder and another infinite on inner cylinder and on the other hand by the radiative flux proportionality to T^4.

Figure 6 shows the time variations of streamline and temperature contours for a concentric configuration with $Ra = 10^5$ and Rc = 1. A secondary cell appears, at $t = t_b \approx 0.035$, on the top of the inner cylinder. Without radiation (Rc = 0), the flow is unicellular for each instant. For $t < t_b$, the position of the principal core moves upward when t increases. Beyond this instant, the secondary cell grows and pushes downward on this core. At t = ts, the intensity

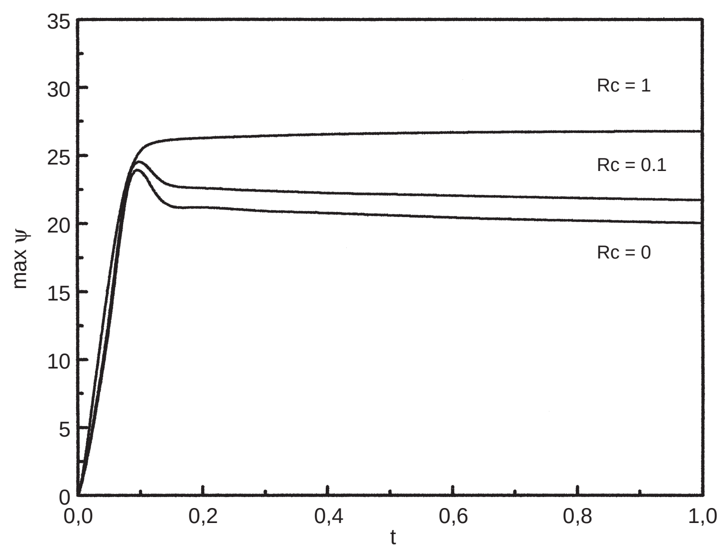


Figure 4.
Influence of the radiation-conduction parameter on the maximum of the stream function for e = –0.625 and $Ra = 10^4$

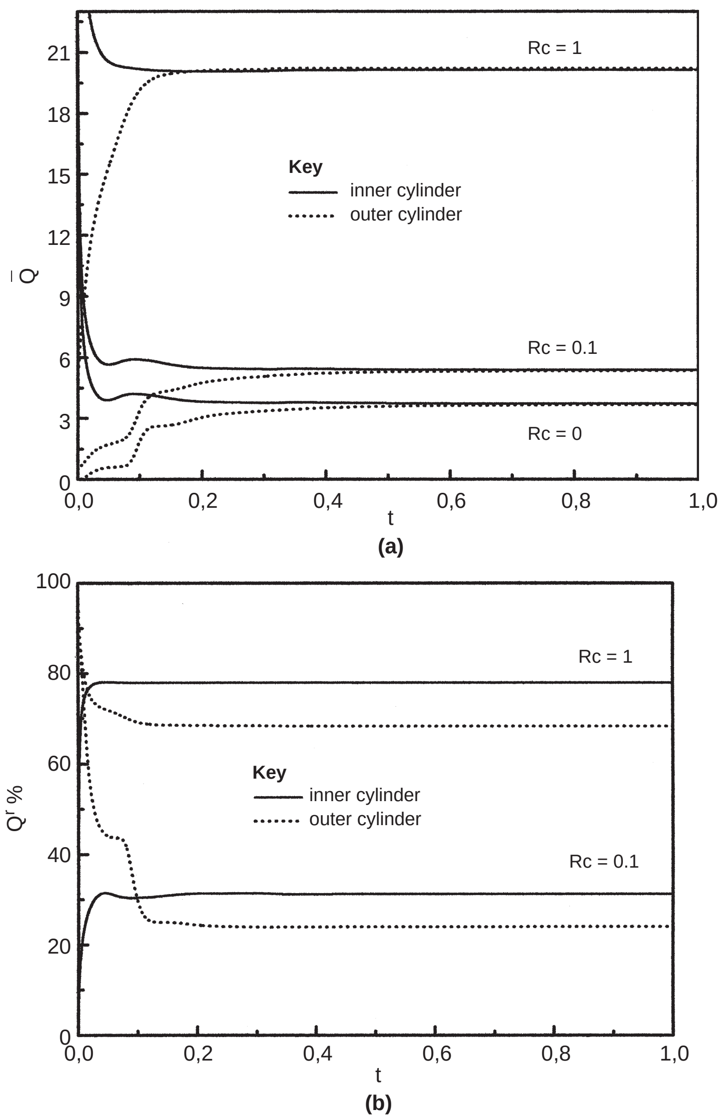
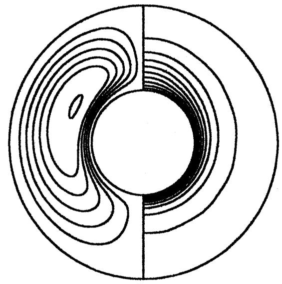


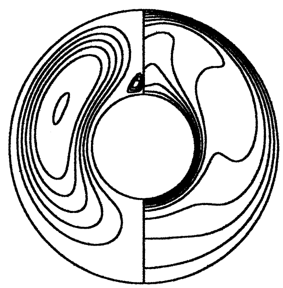
Figure 5.
Influence of the radiation-conduction parameter for e = –0.625 and Ra = 10^4 on: (a) average total inner ($\bar{Q}_{in}$) and outer ($\Gamma \times \bar{Q}_{out}$) fluxes; (b) percentage of the average radiative flux on inner and outer cylinders

of the secondary flow represents 52 per cent of the one of the principal flow. For the same set of parameters, Tan and Howell (1989) noted the presence, in unsteady regime, of this secondary flow.

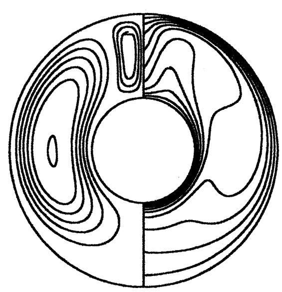
Figure 7a illustrates the time evolution of average total flux on concentric inner and outer cylinders for Ra = 10^5, Rc = 0, 0.1 and 1. For Rc = 1, the inner flux does not vary as regularly as the outer flux due to the formation of a secondary cell just above the inner cylinder. The stationary average total flux decreases by 76 per cent when Rc varies from 1 to 0. According to Figure 7b,



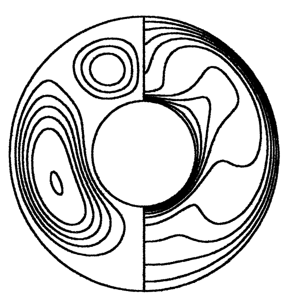
t = 0.02, max(ψ) = 23



t = 0.04, max(ψ) = 62.96, min(ψ) = –0.92



t = 0.06, max(ψ) = 61.75, min(ψ) = –7.88



ts = 0.54, max(ψ) = 50.48, min(ψ) = –26.48

Figure 6.
Time effect on streamline and temperature contours for $e = 0$, $Ra = 10^5$, and $Rc = 1$

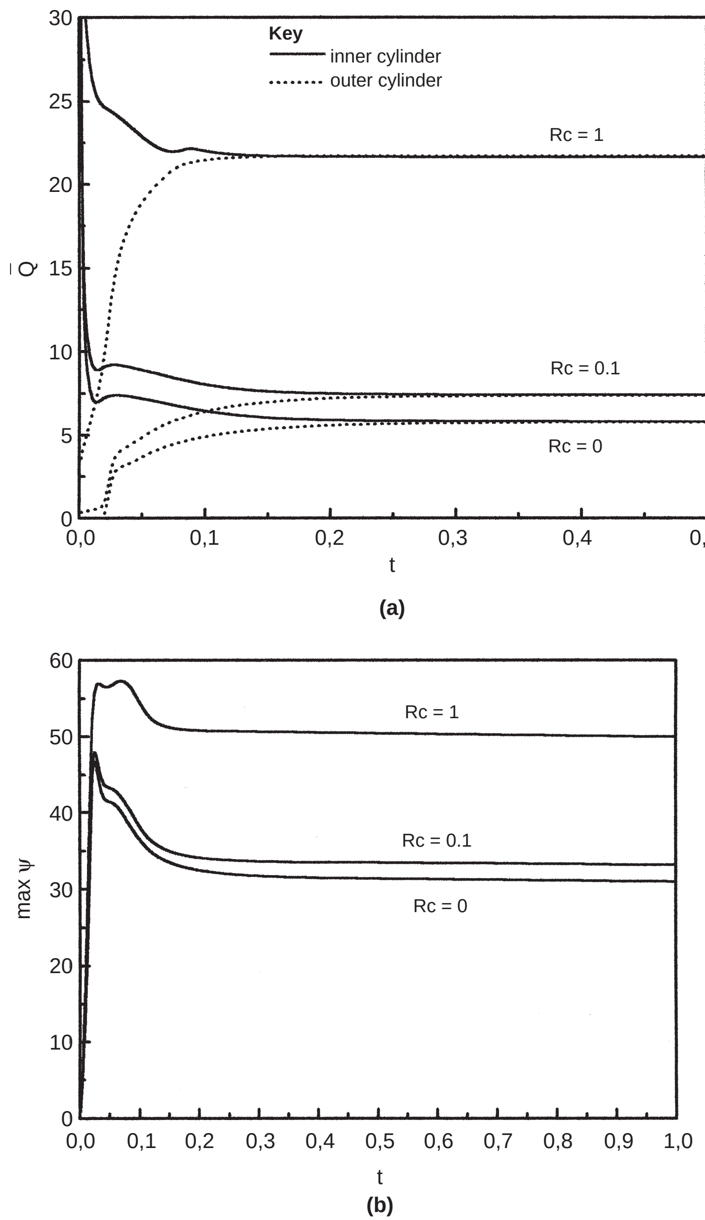


Figure 7. Influence of the radiation-conduction parameter for e = 0 and Ra = 10^5 on: (a) average total inner ($\bar{Q}_{in}$) and outer ($\Gamma \times \bar{Q}_{out}$) fluxes; (b) the maximum of the stream function

the peak presented by the curve is less pronounced for Rc = 1 than for Rc = 0 and 0.1. This is due to the increase of the steady limit of the stream function which rises by 61 per cent when Rc varies from 0 to 1.

For this configuration the calculus is convergent for all values of Rc, while with steady codes (Tan and Howell, 1989; Borjini *et al.*, 1998) no stable solution exists for higher Rc values. Figure 8 shows the disappearance of the secondary flow when Rc rises. For Rc = ∞, the flow presents a symmetry relatively to the horizontal diameter.

Figure 9 shows the evolution in time of streamline and temperature contours for positive vertical eccentricity (e = 0.625) when $Ra = 10^5$ and Rc = 0. A secondary cell originates, at $t = t_b \approx 0.052$, in the proximity of the top of outer cylinder and not near the one of the inner cylinder (concentric configuration,

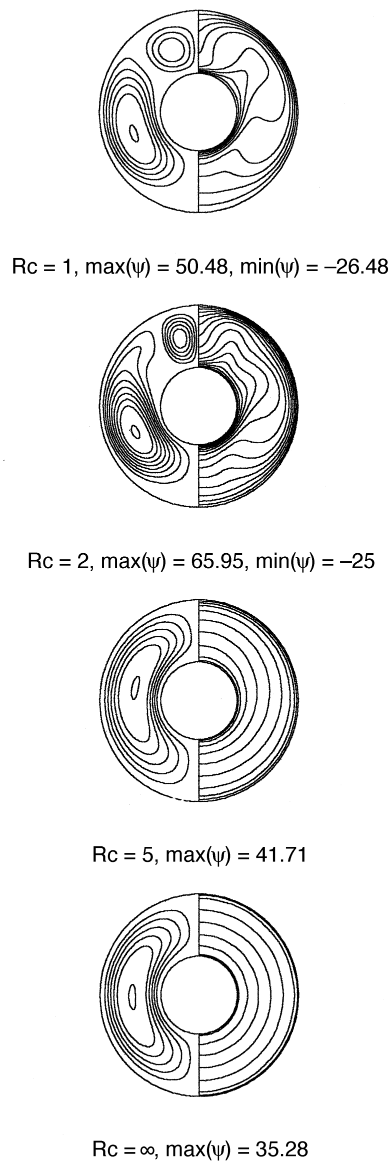
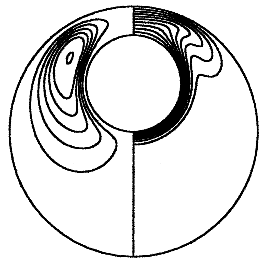
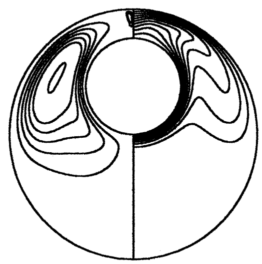


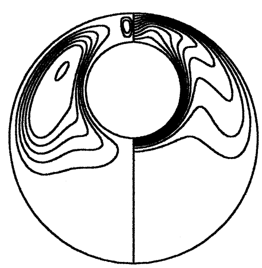
Figure 8.
Influence of the radiation-conduction parameter on streamline and temperature contours for e = 0 and Ra = 10^5



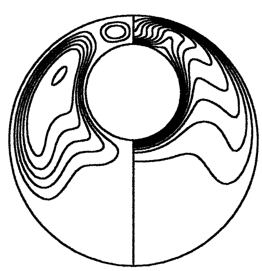
t = 0.02, max(ψ) = 25.33



t = 0.06, max(ψ) = 21.19, min(ψ) = –0.048



t = 0.08, max(ψ) = 19.83, min(ψ) = –0.66



ts = 0.85, max(ψ) = 18.56, min(ψ) = –5.76

Figure 9.
Time effect on streamline and temperature contours for $e = 0.625$, $Ra = 10^5$, and $Rc = 0$

Figure 6). At t = ts, the secondary flow represents 31 per cent of the principal one. The same secondary flow has been founded by Projahn *et al.* (1981) in the steady state. For Rc = 1 and 0.1, the flow is unicellular for each instant. This is confirmed by Figure 10a which shows that, when Rc rises, the maximum of $\max\psi(t)$ becomes less pronounced and disappears for Rc = 1. The steady

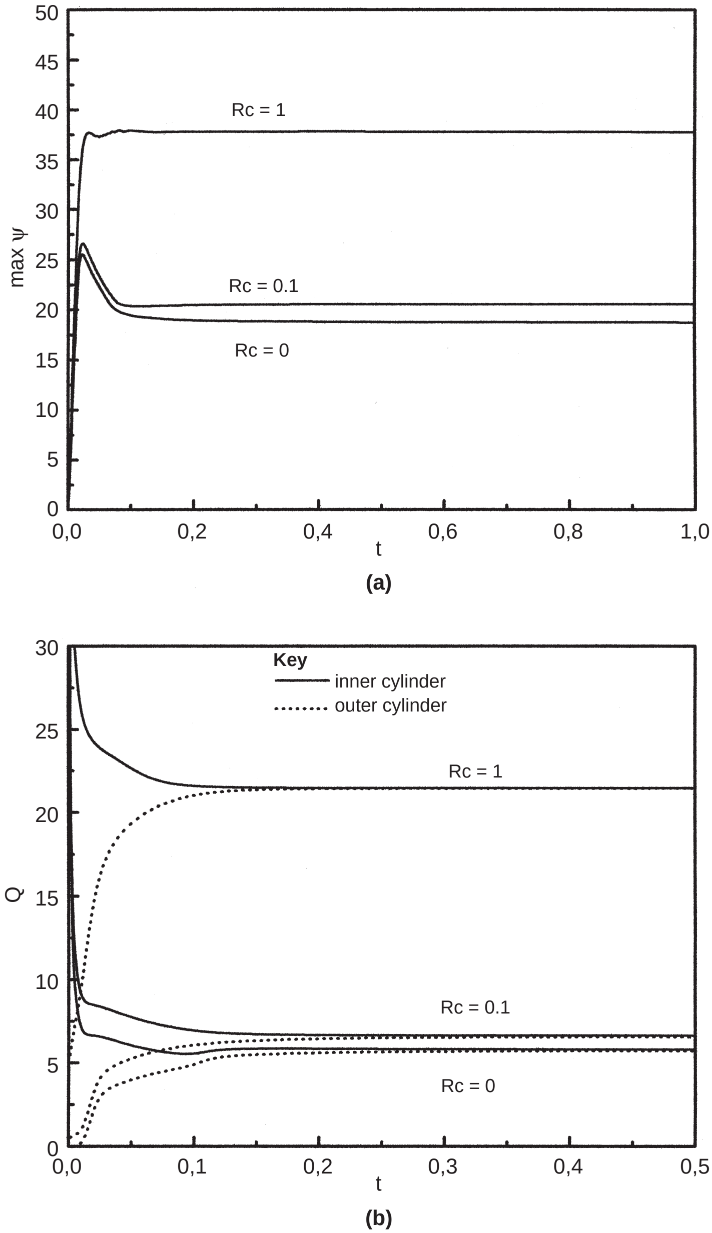


Figure 10. Influence of the radiation-conduction parmeter for e = 0.625 and Ra = 10^5 on: (a) the maximum of the stream function; (b) average total inner ($\bar{Q}_{in}$) and outer ($\Gamma \times \bar{Q}_{out}$) fluxes

maximum of the stream function doubles when Rc varies from 0 to 1. On Figure 10b are presented the time variations of average total fluxes on inner and outer cylinder. For Rc = 1, these variations are regular. When Rc decreases from 1 to 0, the steady heat transfer diminishes by 75 per cent.

Effect of radiative parameters

In this section, computations were performed with e = –0.625, Rc = 1, and $\mathrm{Ra} = 10^4$.

Figure 11 brings to the fore the influence of the wall emissivity on conductive and radiative transfers on both cylinders for $\tau = 1$ and $\omega_0 = 0$. When ε varies from 1 to 0.1, the steady inner and outer radiative fluxes decrease respectively by 90 per cent and 92 per cent. For this variation of ε, the stationary conductive fluxes increase by 58 per cent and 18 per cent on the inner and outer cylinders respectively. Indeed, when the wall emissivity rises, the intensity emitted by the hot cylinder becomes important. This increases radiative flux that heats besides the fluid in the core of the annular space and so diminishes the temperature gradient near the walls. One can note, for $\varepsilon = 0.1$ and 0.5, that the steady conductive fluxes on outer cylinder superpose, while they manifest two distinct transient paths. In the stationary state, the maximum of the stream function rises by 35 per cent when ε increases from 0.1 to 1: it varies from 19.92 to 26.94.

On Figure 12 are presented the time variations of average conductive and radiative fluxes on the inner and outer cylinders for $\varepsilon = 1$, $\omega_0 = 0$, and for different optical thicknesses. When τ varies from 0.1 to 10, the inner and outer steady radiative fluxes reduce respectively by 78 per cent and 88 per cent. The more optically thin the medium is, the more regularly varies the radiative heat transfer. The conductive flux is remarkably less sensitive to τ variation. When τ increases from 0.1 to 1 the steady conductive flux on outer cylinder rises by 61 per cent then still almost constant when τ rises again. For τ = 10, radiative and conductive flux manifest similar profiles. Indeed, for optically thick media, radiation can be assimilated to a diffusion mode of heat transfer with thermal dependent conductivity (Siegel and Howell, 1992). In the steady state, the maximum of the stream function rises by 36 per cent when τ varies from 0.1 to 10: it changes from 21.67 to 29.49. The results of Tan and Howell (1991), concerning a rectangular geometry, show also the sensibility of steady heat transfer and flow to the optical thickness and the wall emissivity.

As shown in Figure 13, the scattering albedo affects little the heat transfer on inner cylinder. When ω_0 varies from 0 to 0.9, the asymptotic limit of outer conductive flux diminishes by 34 per cent. The more diffusing the semi-transparent medium is, the less regularly this flux varies in the transient zone. For the same variation of ω_0, the steady radiative flux on the outer cylinder rises by 11 per cent and the steady maximum of the stream function reduces by 16 per cent: it changes from 26.94 to 22.72.

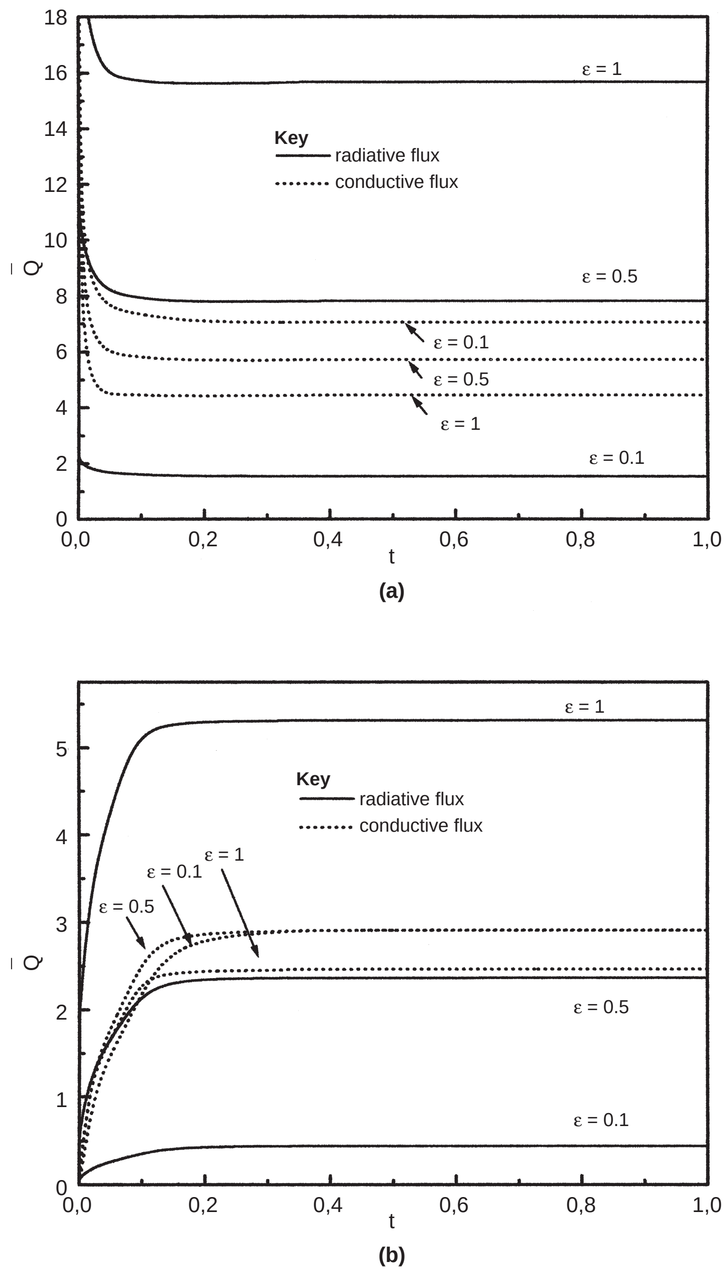


Figure 11. Influence of wall emissivity on conductive and radiative average fluxes when e = –0.625, Ra = 10^4, Rc =1, τ = 1, and ω_0 = 0 (a) inner cylinder; (b) outer cylinder

Conclusion

Using the finite volume method to resolve the radiative transfer equation and control volume Patankar's technique, we have analysed the effect of radiative transfer on unsteady natural convection in horizontal annular space.

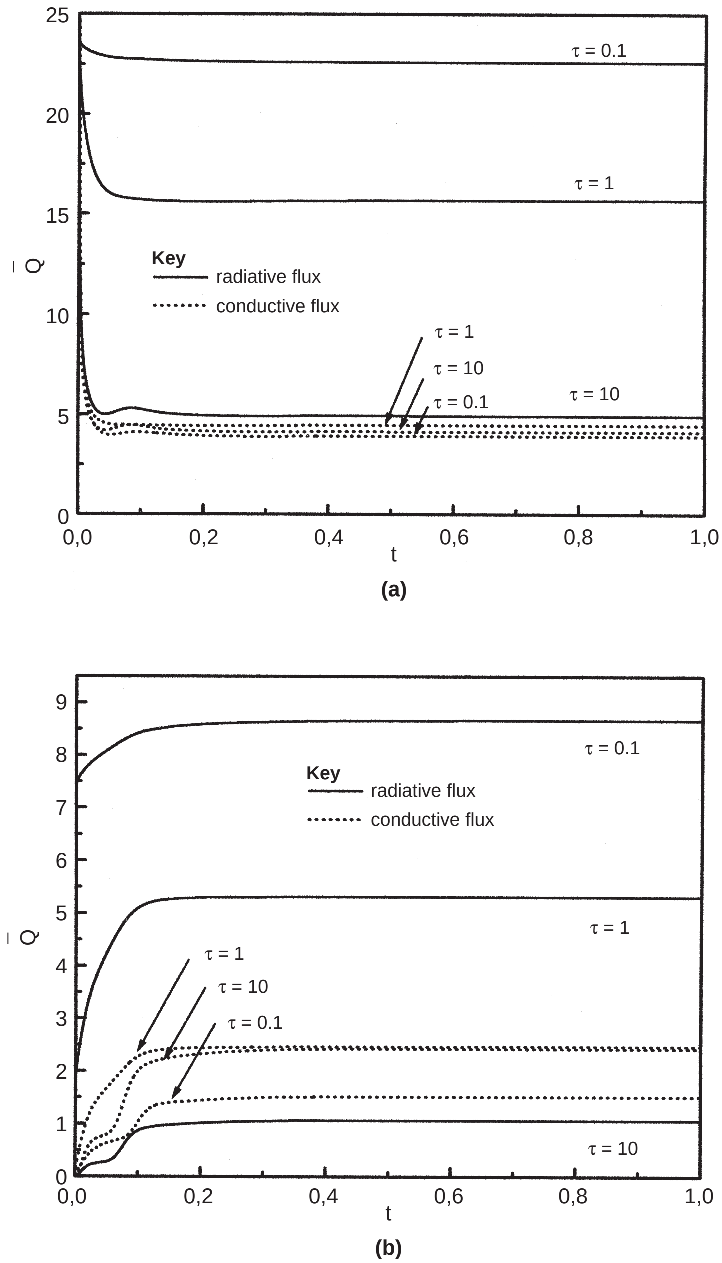


Figure 12.
Influence of optical thickness on conductive and radiative average fluxes, when e = –0.625, Ra = 10^4, Rc = 1, $\varepsilon = 1$, and $\omega_0 = 0$
(a) inner cylinder;
(b) outer cylinder

The effect of radiative transfer is as important in the steady zone as in the unsteady one. The increase of radiation-conduction parameter, in the absence of any generation of secondary flow, smoothes the heat flux profiles in the unsteady region and accelerates the internal temperature balancing.

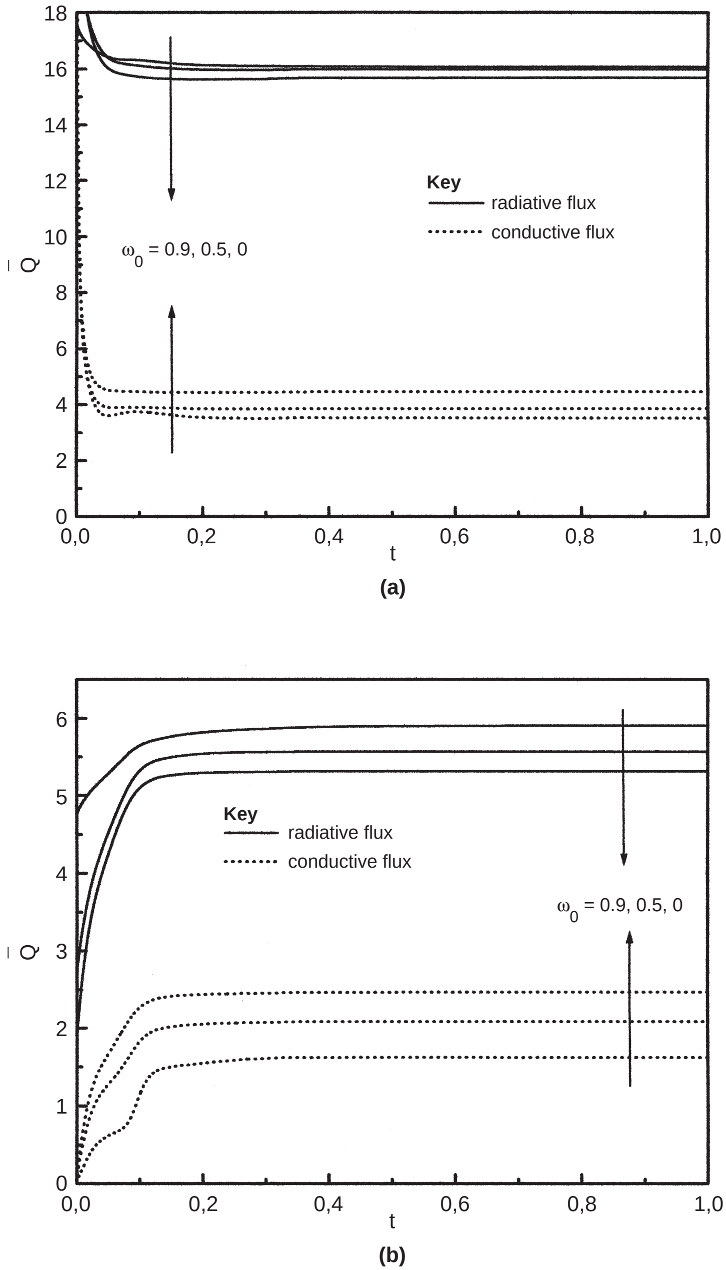


Figure 13.
Influence of scattering albedo on conductive and radiative average fluxes when e = –0.625, Ra = 10^4, Rc = 1, ε = 1, and τ = 1
(a) inner cylinder;
(b) outer cylinder

In the steady zone, radiative heat transfer is more influent on the inner cylinder than on the outer cylinder and conversely in the transient zone. When t increases, this influence diminishes on the outer cylinder and accentuates on the inner cylinder.

The transient zone is more narrow in the presence of radiation than in pure convection. This can justify the use of steady coefficient of heat transfer to model unsteady systems.

In pure convection, the secondary flow originates near the top of outer cylinder (cold) and near the one of the inner cylinder (hot) when the medium is radiatively participant.

The optical thickness and the wall emissivity affect considerably heat transfer and the intensity of the flow. The more optically thin the medium is, the more regular are the time variations of radiative transfer. The scattering albedo influences more the outer cylinder than the inner one. The more diffusing the medium is, the less regular are the variations of unsteady conductive fluxes.

In some cases, there is a discordance between the results obtained from steady codes and from this unsteady code.

References

Bejan, A. (1984), *Convection Heat Transfer*, John Wiley & Sons, New York, NY.

Borjini, M.N., Mbow, C. and Daguenet, M. (1998), "Etude numérique de l'influence du transfert radiatif sur la convection naturelle laminaire, bidimensionnelle, permanente, dans un espace annulaire d'axe horizontal, délimité par deux cylindres circulaires isothermes", *Rev. Gén. Ther.*, Vol. 37, pp. 475-87.

Chai, J.C., Lee, H.S. and Patankar, S.V. (1994), "Finite volume method for radiative heat transfer", *J. Thermophysics and Heat Transfer*, Vol. 8, pp. 419-25.

Chai, J.C., Parthasarathy, G., Lee, H.S. and Patankar, S.V. (1995), "Finite volume radiative heat transfer procedure for irregular geometries", *J. Thermophysics and Heat Transfer*, Vol. 9, pp. 410-15.

Chang, L.C., Yang, K.T. and Lioyd, J.R. (1983), "Radiation-natural convection interactions in two-dimensional complex enclosures", *J. Heat Transfer*, Vol. 105, pp. 89-95.

Charrier-Mojtabi, M.C., Mojtabi, A. and Caltagirone, J.P. (1979), "Numerical solution of a flow due to natural convection in horizontal cylindrical annulus", *J. Heat Transfer*, Vol. 101, pp. 171-3.

Chui, E.H., Raithby, G.D. and Hughes, P.M.J. (1992), "Prediction of radiative transfer in cylindrical enclosures with the finite volume method", *J. Thermophysics and Heat Transfer*, Vol. 6, pp. 605-11.

Fusegi, T. and Farouk, B. (1989), "Laminar and turbulent natural convection-radiation interactions in a square enclosure filled with a nongray gas", *Numerical Heat Transfer*, Vol. 15, pp. 303-22.

Hassab, M.A. and Özizik, M.N. (1979), " Effects of radiation and convective boundary conditions on the stability of the fluid in an inclined slender slot", *Int. J. Heat Mass Transfer*, Vol. 22, pp. 1095-105.

Karki, K.C. and Patankar, S.V. (1988), "Solution of some two-dimensional incompressible flow problems using a curvilinear coordinate system based calculation procedure", *Numerical Heat Transfer*, Vol. 14, pp. 309-21.

Kim, T.Y. and Baek, S.W. (1991), "Analysis of combined conductive and radiative heat transfer in a two-dimensional rectangular enclosure using the discrete ordinates method", *Int. J. Heat Mass Transfer*, Vol. 34, pp. 2265-73.

Kim, T.Y. and Baek, S.W. (1997), "Analysis of radiative transfer in cylindrical enclosures using the finite volume method", *J. Thermophysics Heat Transfer*, Vol. 11, pp. 246-52.

Küblbeck, K., Merker, G.P. and Straub, J. (1980), "Advanced numerical computation of two-dimensional time-dependent free convection in cavities", *Int. J. Heat Mass Transfer*, Vol. 23, pp. 203-17.

Kuehn, T.H. and Goldstein, R.J. (1976), "An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders", *J. Fluid Mech.*, Vol. 74, pp. 695-719.

Kuehn, T.H. and Goldstein, R.J. (1978), "An experimental study of natural convection heat transfer in concentric and eccentric horizontal cylindrical annuli", *J. Heat Transfer*, Vol. 100, pp. 635-40.

Kumar, R. (1988), "Study of natural convection in horizontal annuli", *Int. J. Heat Mass Transfer*, Vol. 31, pp. 1137-48.

Larson, D.W. and Viskanta, R. (1976), "Transient combined laminar free convection and radiation in a rectangular enclosure", *J. Fluid Mech.*, Vol. 78, pp. 65-85.

Lauriat, G. and Desreyaud, G. (1985), "Influences of the boundary coditions and linearization on the stability of a radiating fluid in a vertical layer", *Int. J. Heat Mass Transfer*, Vol. 28, pp. 1613-17.

Lee, K.H., Lee, J.S. and Choi, M. (1996), "Parametric analysis of radiative-convective heat transfer around a circular cylinder in a cross flow using the finite volume radiation solution method", *Numerical Heat Transfer*, Part A, Vol. 29, pp. 181-96.

Moder, J.P., Chai, J.C., Parthasarthy, G., Lee, H.S. and Patankar, S.V. (1996), "Nonaxisymmetric radiative transfer in cylindrical enclosures", *Numerical Heat Transfer*, Part B, Fundamentals, Vol. 30, pp. 437-52.

Moon, P. and Spencer, D.E. (1971), *Field Theory Handbook*, Springer, Berlin.

Onyegegbu, S.O. (1986), "Heat transfer inside a horizontal cylindrical annulus in the presence of thermal radiation and buoyancy", *Int. J. Heat Mass Transfer*, Vol. 29, pp. 659-71.

Patankar, S.V. (1980), "Numerical heat transfer and fluid flow", McGraw-Hill, New York, NY.

Powe, R.E. , Carley, C.T. and Carruth, S.L. (1971), "A numerical solution for natural convection in cylindrical annuli", *J. Heat Transfer*, Vol. 92, pp. 210-20.

Projahn, U., Rieger, H. and Beer, H. (1981), "Numerical analysis of laminar natural convection between concentric and eccentric cylinders", *Numerical Heat Transfer*, Vol. 4, pp. 131-46.

Raithby, G.D. and Chui, E.H. (1990), "A finite volume method for predicting a radiant heat transfer in enclosures with participating media", *J. Heat Transfer*, Vol. 112, pp. 415-23.

Siegel, R. and Howell, J.R. (1992), *Thermal Radiation Heat Transfer*, 3rd ed., Hemisphere Publishing, New York, NY.

Tan, Z. and Howell, J.R (1989), "Combined radiation and natural convection in a participating medium between horizontal concentric cylinders", in *National Heat Transfer Conference, Heat Transfer Phenomena in Radiation, Combustion and Fires*, Vol. 106, pp. 87-94.

Tan, Z. and Howell, J. R. (1991) "Combined radiation and natural convection in a two-dimensional participating square medium", *Int. J. Heat Mass Transfer*, Vol. 34, pp. 785-93.

Tsui, Y.T. and Tremblay, B. (1984), "On transient natural convection heat transfer in the annulus between concentric, horizontal cylinders with isothermal surfaces", *Int. J. Heat Mass Transfer*, Vol. 27, pp. 103-11.

Webb, B.W. and Viskanta, R. (1987), "Radiation-induced buoyancy-driven flow in rectangular enclosures: experiment and analysis", *J. Heat Transfer*, Vol. 109, pp. 427-33.

Yucel, A., Acharya, S. and Williams, M.L. (1989), "Natural convection and radiation in a square enclosure", *Numerical Heat Transfer*, Vol. 15, pp. 261-77.